FORM ANALYSIS.

By A. A. MICHELSON.

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As a recreation in the midst of more serious work, I have been interested in the analysis of natural forms; and hoping that the results of this somewhat desultory occupation be not deemed too frivolous for so august an occasion, I will venture to present some illustrations[1] and generalizations which have occurred to me. I recognize that the subject is one whose adequate treatment would tax the best efforts of one who combined the insight of the scientist, with the æsthetic appreciation of the painter and the gift of language of the poet—and certainly I am lacking in all three—but especially in the power of adequate expression.

I had hoped that my contribution would at least have the merit of originality, but I find that many abler investigators have found a similar delight in this interesting field, and have expounded their ideas with a wealth of poetic imagery and of exquisite illustration such as I cannot hope to emulate.

Haeckel in particular has treated the subject of General Morphology in so exhaustive a fashion that it would seem futile to attempt to add anything of real interest or importance. This is most assuredly the case as regards luxurious wealth of illustration, and better cannot be done than to employ the "Art Forms in Nature" with grateful thanks to the man who has placed these within our reach.

Regarding his classification of forms there may be room for differences of opinion; and here I may venture to suggest some modifications for such a general scheme as shall include not only natural objects but also physical and mathematical forms such as interference patterns and graphs of analytical expressions.

[1] The paper was illustrated with lantern slides taken mainly from Haeckel's "Kunstformen der Natur."

Classification of Forms.

I. *Symmetrical.*

II. *Unsymmetrical.*

I. Symmetrical forms.

Congruence of parts by

A. Rotation through 180° (Odd Symmetry).

B. Reflection in plane (Even Symmetry).

C. Rotation + Reflection (Partial Symmetry).

1. Radial Symmetry. (Corresponding points equidistant from radiant.)
2. Axial Symmetry. (Corresponding points equidistant from a fiducial line.)
3. Plane Symmetry. (Corresponding points equidistant from plane.)

1. Radial Symmetry.
   - *a. Central* Symmetry. (Corresponding points on straight line through center.)
   - *b. Ovoid* Symmetry. (Radiant in axis but not central.)
   - *c. Excentric* Symmetry. (Radiant not in an axis.)
2. Axial Symmetry.
   - *a. Centraxal* Symmetry. (Corresponding points on same perpendicular through axis.)
   - *b. Dorsiventral* Symmetry. (Corresponding points not on same perpendicular.)
3. Plane Symmetry.
   - *a. Triplanar* Symmetry.
   - *b. Biplanar* Symmetry.
   - *c. Bilateral* Symmetry.

The foregoing general systems of symmetry might be carried out in greater detail. For instance, the central symmetrical forms might be subdivided into (1) Spherical, (2) spheroidal, (3) ellipsoidal, (4) monoclinal, and (5) triclinal. So also the triplanar symmetrical forms may be classified into (1) Isometric, (2) quadratic, (3) rhombic; and the centraxial forms according to the number of similar parts into which an equatorial section may be divided, ($n = 2, 3, 4, 5 \ldots \infty$).

This, however, may well be left open, so that specialists in the various branches of science may elaborate details in such a way as to best fit their respective needs.

It will be noted that the proposed system extends the idea of radial symmetry to include the vast array of natural forms which *radiate,* though not necessarily from a center of figure and which would otherwise be considered either unsymmetrical, or as having merely axial or plane symmetry.

Another point is the *explicit* recognition of "odd" symmetry. This kind of symmetry is admitted by the mathematician on the same terms as "even"—though not so generally by the biologist or the crystallographer. It is easy to show that the one is quite as logical as the other.

The definition of symmetry is necessarily arbitrary—as in fact is well shown by the varied significance attached to the term by biologists, crystallographers and mathematicians. The last are coming to recognize any form as symmetrical when any admitted essential character is unaltered by any specified operation. So broad a definition would scarcely emphasize what is generally understood by the term; but if the "operations" be restricted to (1) rotation through two right angles, and (2) reflection in a plane, a symmetrical form may be defined as one whose corresponding points are (*a*) equidistant from a point, a line, or a plane; and (*b*) in which congruence may be obtained by a single "operation."

If condition *a* is fulfilled, but not *b* (as in the case of triclinic crystals), or if more than one operation is required for congruence, the form may be said to have *partial symmetry.*[1]

II. Unsymmetrical Forms.

Forms exist in endless variety which do not fall under any of these divisions, whose beauty and interest may be fully as striking as the symmetrical forms.

These may be classified according to the following scheme, which is only provisional.

A. Rhythmic.

[1] If it were not for condition *a* the helix would have to be considered a symmetrical form; but *b* provides that it have partial symmetry.

B. Spiral.

C. Branching.

D. Scattered.

E. Irregular.

It would far exceed the limits of time and space to develop these in detail, but it may be mentioned that:

Rhythmic forms may appropriately be divided into *Line* rhythms, *Nets* or surface rhythms, and *Space* rhythms.

Further the "element" of the rhythm may be constant or (regularly) variable, and (in the case of linear rhythms) the line may be straight or curved.

Spirals may include flat, cylindrical or conoidal forms.

Branching forms may include all the well-known systems of arrangement of leaves and flowers, as well as the ordinary types of branches, to which may be added a number of arrangements peculiar to some of the lower organisms.

Scattered forms are such as are typified by the arrangement of the stars in space.

Irregular forms may be subdivided into Definite, Indefinite, Distorted.

In many cases the essential character of both symmetrical and unsymmetrical forms may be expressed in mathematical language; and even in the cases where such formulæ are only approximate expressions of the reality, such expressions may be the best and readiest means of ascertaining departures from the law.

Doubtless the biologist will look askance at what appears a grewsome array of mathematical symbols—at least until he finds how much less vicious they are than they look—but for the student of mathematics, there can scarcely be a more interesting and instructive practice for familiarizing himself with the properties of analytical functions, than to exercise his ingenuity in fitting the most appropriate expression to the forms he sees in Nature.

A further possibility in the expression of natural forms in mathematical language is the presentation of an ideal—so to speak—which the object is trying to embody; and the attempt to determine what this form would be under standard conditions would furnish an interesting and fruitful subject for research.

In contemplating the variety of exquisitely graceful forms which are exhibited in such abundance by the lower organisms, it is difficult to realize that their building up may be the result of the action of purely physical causes—and in truth it may be long before the theory of their formation is complete.

It is safe to say, however, that such general characters as symmetry of parts, rhythmic arrangements, systems of branching—which occur in strikingly similar fashion in such widely dissimilar objects as vegetation, protozoa, crytals, even liquids—must have some very general and fundamental explanation.

In many of the simpler cases such explanations are almost self-evident. For instance, the very frequent occurrence of spherical or spheroidal forms may be accounted for by the action of capillary forces; and the regular segmentation figures shown in many protozoan cells may be explained by the geometrical necessities of the case. An experimental study of such geometrical relations for figures in one plane was published by Alfred Mayer, and his results furnish many suggestive analogies which bear on the structure of the molecule as well as that of the micro-organism. An extension of the results to three dimensions would doubtless encounter serious experimental difficulties; but these may be overcome, or at least reduced sufficiently for practical purposes. The problem is really however a geometrical one—though the figures resulting from a purely mathematical investigation would be complicated by considerations of stability.

As an instance in which the cause of regularity is reasonably certain, let us consider the forms produced by a drop of colored liquid falling into a body of liquid of nearly the same density. As the drop descends it flattens out, the edge advancing more rapidly than the flattened surface. The equilibrium, already unstable, is still further disturbed by the momentum acquired by the peripheral parts, which causes them to assume a bell-shape. This form is still unstable, and if any accidental cause starts a thickening into drops, this will be assisted by the thinning of the adjacent parts which then lag behind. The same forces act at other points on the bell, to form "drops," the distance from the other drops being just enough to save being thinned. This distance (which depends on

the properties of the liquid) determines the "period" of the rhythm. The drops or loops once formed produce vortex motions in the adjacent liquid which tend to separate them, and the action of these quasi repulsions is the stronger the closer the drops are to each other. These will consequently separate until the "repulsions" are equal—that is, till they are equidistant.

The same general explanation holds if the original drop is transformed into a ring.

The result is a beautifully regular (but inverted) dichotomous (tri- or tetrachotomous) branching, closely resembling vegetation.

The study of symmetry is doubtless of relatively subordinate importance from the point of view of the specialist—though Haeckel rightly insists on its significance in the evolution of the lower organisms.

In the study of crystallography considerations of symmetry are of high value; and a clear conception of relations of symmetry often wonderfully simplify the most abstruse problems in mathematical physics.

In the construction of buildings and bridges and engineering works in general as well as in machinery and scientific apparatus, and even of common tools and utensils,—the very necessities of the case have enforced a more or less complete symmetry of parts and of the whole.

Indeed, it is a common experience that the design of a piece of machinery may be so altered as to make it symmetrical often with a surprising increase in efficiency as well as beauty.

In designing for the sake of decoration, symmetrical forms are everywhere manifest, and the perception of their mutual relations is indispensable to the student of art. Occasionally, however, there is in decoration a deliberate departure from symmetry, and such a variation may greatly enhance the beauty and effectiveness of the design. We tire of too great uniformity even of agreeable kinds, and the element of variety is as important in art as an occasional discord is in music—its purpose being to heighten the effect of the succeeding harmony.

One of the great disadvantages of the modern tendency to extreme specialization in research is the loss of companionship of

the sister sciences, with the attendant loss of perspective which a more general survey of the whole field of science should furnish. Should we not, then, utilize every opportunity which promises to further their union?

The geologist, the chemist, the physicist, the mathematician, may and occasionally do meet here on the common ground of crystallography. By a comparatively slight extension, the "ground forms" of organisms—as Haeckel terms them—may also be included with a corresponding extension of our society of sciences to include zoölogy and botany.

Nay, Art will demand a chair at the banquet, and Music and Poetry will also grace the feast.